2014 Specimen Paper 2 Solutions

✓ (1) a. Mode : 47.5 cm (because it has the highest frequency)

✓✓ b. i. mean : 45.85 cm (or 45.9)

✓ ii. std. deviation 17.1 cm

✓ iii. median = 47.5 cm

Since the median would be the average of the values in 50th and 51st position.

$\bar{x} = \frac{\sum f \cdot x}{n}$

$= \frac{4585}{100}$

$= 45.85$

using $\bar{x} = 45.9$

$S_x = \sqrt{\frac{f \cdot (x_i - \bar{x})^2}{n}}$

$= \sqrt{\frac{29203}{100}} = 17.1$

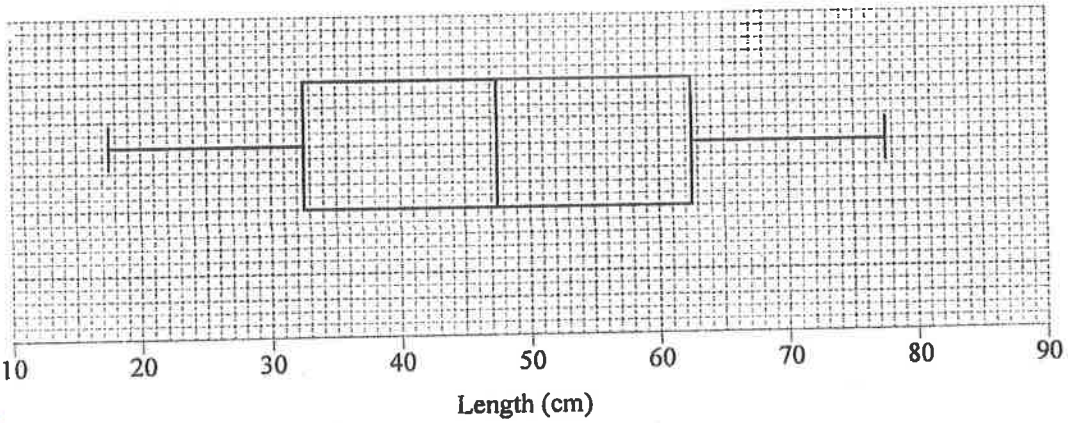
can use GDC lists

✓✓ c. Q1 (avg of values of 25th and 26th position) 32.5

Q3 (" " " " 75th and 76th ") 62.5

$\therefore$ IQR = 62.5 − 32.5 = 30 cm

✓✓ d.



Scoring

(A1) for correct label and scale
(A1)(ft) for correct median
(A1)(ft) for correct quartiles and box
(A1) for endpoints at 17.5 and 77.5 joined to box by straight lines *(A1)(A1)*(ft)*(A1)*(ft)*(A1)* ***[4 marks]***

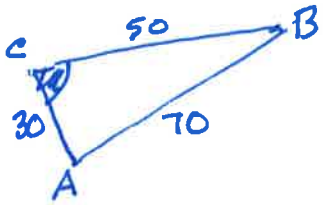
the final point is lost if the lines go through the box.

✓✓ e. $\epsilon = \left| \frac{43 - 45.85}{45.85} \right| \times 100\% = 6.22\%$ FT

13 total

2.

✓✓ (a)

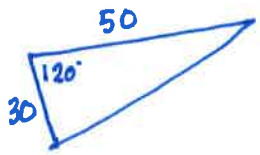


Law of Cosines

$\cos ACB = \dfrac{30^2 + 50^2 - 70^2}{2 \cdot 30 \cdot 50} = -.5$

$\boxed{ACB = 120^\circ}$

✓✓ (b)



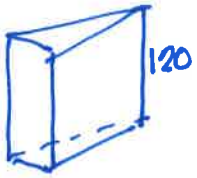
SAS Area formula $A = \frac{1}{2}(50 \cdot 30 \cdot \sin 120^\circ)$

$= 649.519\ldots = \boxed{650\ m^2}$

UNITS required

FT

✓✓ (c)

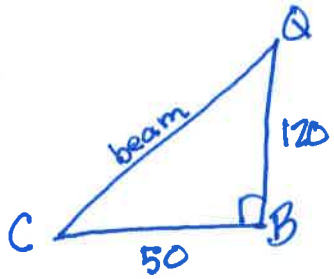


Volume = area of base × height

$= (649.519\ldots)(120) = 7794.2 = \boxed{77900\ m^3}$ 3 sigfigs

Also accepted: $78\,000\ m^3$ if $650\ m^2$ was used.

✓✓ (d)

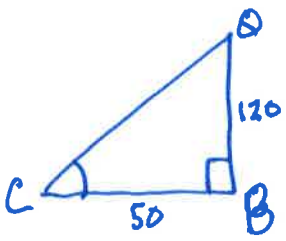


$CQ^2 = 50^2 + 120^2$

Pythag. theorem

$\boxed{CQ = 130\ m}$ UNITS NOT required

✓✓ (e)



can use Soh-Cah-Toa

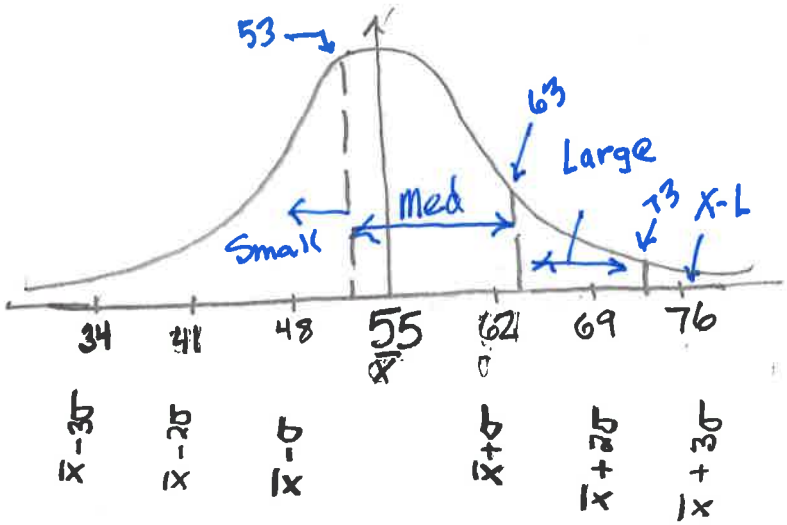
$\tan QCB = \dfrac{120}{50}$

$QCB = 67.3801\ldots = \boxed{67.4^\circ}$

12 total

3 $\bar{x} = 55g$ $\sigma = 7$ Normally Distributed

a)



A1 for normal curve with mean of 55

A1 3 lines in approx. correct position

A1 labels shown for 53, 63, and 73

b) (i) $P(\text{medium}) = P(53 \leq \text{Weight} < 63) = .485902\ldots = \boxed{.486}$

the answer can be calculated with GDC using normalcdf(53, 63, 55, 7)

Lower boundary; Upper boundary; $\bar{x}$; σ

don't need to write this

(ii) $P(XL) = P(\text{Weight} > 73) = .00506402\ldots = \boxed{.00506}$

normalcdf(73, 10^{99}, 55, 7)

c) $P(\text{Weight} > w) = 0.3$

$w = 58.6708\ldots$ $\boxed{58.7}$

invNorm(.7, 55, 7)

must be area to left of mark

so $1 - 0.3 = 0.7$

d) Expected number of large eggs

$= 2000(.121)$

$= 242$ large eggs

continues ↓

3e)

small	$2000(.388)(.30) =$	$\$232.80$
med	$2000(.486)(.50) =$	486.00
L	$2000(.121)(.65) =$	157.30
XL	$2000(.00506)(.80) =$	8.10
		$\$884.20$

Estimated monthly income

4. $I = k(1.05)^{-d} = k\left(\frac{1}{1.05}\right)^{d}$ when $d = 0$ (at surface) $I = 100\%$

✓✓ a) when $d = 0$ $\quad 100 = k(1.05)^{0}$ so $k = 100$

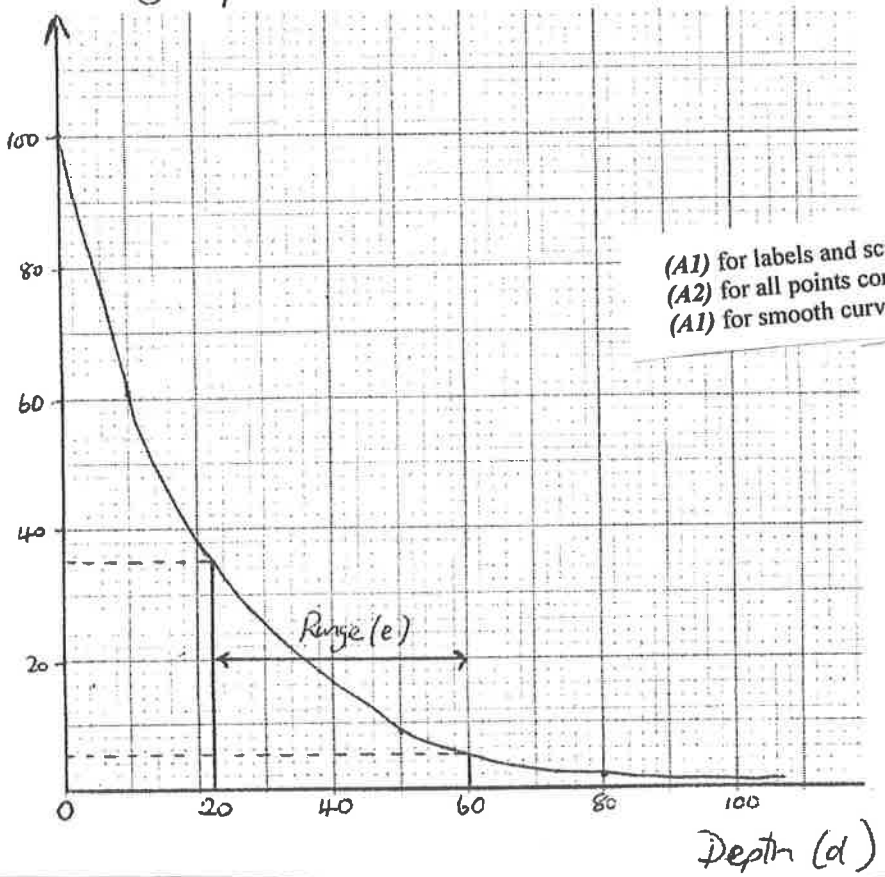
✓✓ b) $I = 100(1.05)^{-25} = 29.5302\ldots$ **29.5 %** FT

✓✓ c) $65 = 100(1.05)^{-d}$

$.65 = \left(\frac{1}{1.05}\right)^{d}$ $\quad \log(.65) = \log\left(\frac{1}{1.05}\right)^{d}$ $\quad d = \frac{\log(.65)}{\log\left(\frac{1}{1.05}\right)}$

$d = 8.82929\ldots = $ **8.83 m**

✓✓ ✓✓ d) Intensity I %



(A1) for labels and scales
(A2) for all points correct, *(A1)* for 3 or 4 points correct
(A1) for smooth curve asymptotic to the *x* axis

✓✓ e) See markings on graph

5% $\quad 5 = 100(1.05)^{-d} \rightarrow d = 61.4$

35% $\quad 35 = 100(1.05)^{-d} \rightarrow d = 21.5$

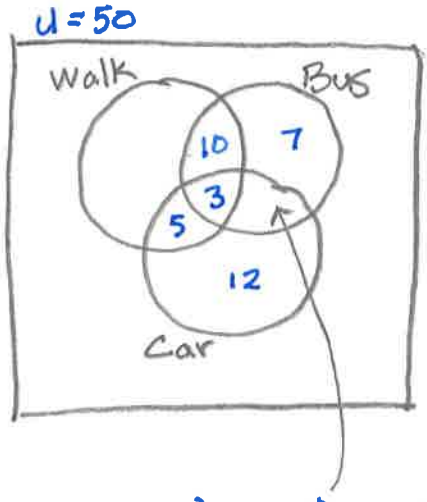
A1 if lines are in approximate position in graph

A1 if range of values indicated with arrows or shading.

12 total

5

a)



Scoring

A1 for rectangle and 3 intersecting circles

A1 for "3" correct

A1 for 5 and 10 correct

A1 for 7 and 12

b). (i) $28 - (10+3+7) = 8$ FT

(ii) $5+3+8+12 = 28$ FT

c) Interpretation - Probability that she walked given that she took the bus

$P(\text{walk} \mid \text{bus}) = \boxed{\frac{13}{28}}$ FT ← reduced sample space

other answers accepted: .464 46.4 %

d)

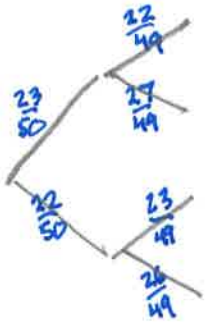
(i) $P(\text{both walked}) = P(\text{1st walked and 2nd walked}) = \frac{23}{50} \times \frac{22}{49}$ M1M1

$= \boxed{\frac{506}{2450}}$

or .207 20.7 %

(ii) Probability only one walked

$= P(\text{1st person walked only or 2nd person walked only})$



$= \frac{23}{50} \times \frac{27}{49} + \frac{27}{50} \times \frac{23}{49}$

$= \boxed{\frac{1242}{2450}}$

or .507 50.7 %

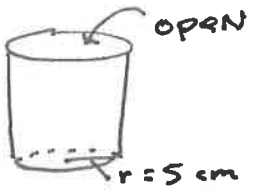
17 total

6

(a) Base (circle)

✓✓ (i) $A = \pi r^2 = \pi (5)^2 = \boxed{78.5 \text{ cm}^2}$

(25π cm² accepted)



$V = 8000 \text{ cm}^3$

✓✓ (ii) V = area of base × height

$8000 = 25\pi \times h$

$h = 101.8591\ldots = \boxed{102 \text{ cm}}$

✓✓✓ (iii) surface area = 1 base + curved Area

$= \pi r^2 + 2\pi r h$

$= \pi (5)^2 + 2\pi (5)(101.8591\ldots)$

$= 3278.5398\ldots$

$= \boxed{3280 \text{ cm}^2}$

would still be 3280 if $h = 102$ was used

✓✓ (b) A height of 102 cm and a radius of 5 cm would be too narrow and tall.

✓ (c) Merryn 8000 cm³ minimize surface area $\boxed{8000 = \pi r^2 h}$

✓✓ (d) TOTAL SURFACE Area (ONE base) $= \pi r^2 + 2\pi r h$

solve for $h = \frac{8000}{\pi r^2}$

$= \pi r^2 + 2\pi r \left(\frac{8000}{\pi r^2}\right)$

$= \pi r^2 + \frac{16000}{r}$

Part d scoring

A1 for correct re-arrangement of part c.

M1 for substitution of this same arrangement into surface area formula.

6e) $A = \pi r^2 + 16000r^{-1}$ so $\frac{dA}{dr} = 2\pi r - 16000r^{-2}$ $= 2\pi r - \frac{16000}{r^2}$

A1 for $2\pi r$
A1 for -16000
A1 for r^{-2}
subtract A1 if extra term is present

f) (i) $\frac{dA}{dr} = 0$ $2\pi r - \frac{16000}{r^2} = 0$

$2\pi r^3 - 16000 = 0$

$2\pi r^3 = 16000$

$r^3 = \frac{16000}{2\pi}$

$r = \sqrt[3]{\frac{16000}{2\pi}} = 13.6556\ldots$

$= 13.7$ cm

(ii) $h = \frac{8000}{\pi(13.6556)^2} = 13.6556\ldots. = 13.7$ cm

13.6 will be accepted also if $r = 13.7$ was used.

g) This design is more reasonable since the height is about the same as the radius

Scoring: Yes or No could be correct as long as a reasonable explanation is given

22 total